Name: ______________________________

Instructor: __________________________

Math 10560, Practice Exam 2.
March 19, 2025

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 12 pages of the test.
- Each multiple choice question is worth 7 points. Your score will be the sum of the best 10 scores on the multiple choice questions plus your score on questions 13-16.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)
11.	(a)	(b)	(c)	(d)	(e)
12.	(a)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice	________
13.	________
14.	________
15.	________
Total	________

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Multiple Choice

1.(7 pts.) Use Simpson's rule with $n = 4$ to appoximate the integral $\int_0^4 f(x)dx$ where a table of values for the function $f(x)$ is given below.

x	0	1	2	3	4
$f(x)$	2	1	2	3	5

(a) 11 (b) 8 (c) 9 (d) 9.5 (e) 10.4

2.(7 pts.) Evaluate the improper integral

$$\int_4^\infty \frac{1}{(x-2)(x-3)}\, dx.$$

(a) the integral diverges (b) $\ln 3$ (c) $\ln \frac{1}{2}$

(d) $\ln 2$ (e) $3 \ln 2$

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3.(7 pts.) What can be said about the integrals

$$(i) \int_0^1 \frac{e^x}{x^2}\,dx;$$

$$(ii) \int_1^\infty \frac{\cos^2 x}{x^2}\,dx?$$

(a) (i) converges and (ii) diverges

(b) both (i) and (ii) diverge

(c) (i) diverges and (ii) converges

(d) neither integral (i) nor (ii) is improper

(e) both (i) and (ii) converge

4.(7 pts.) Which of the following is an expression for the arclength of the curve $y = \cos x$ between $x = \frac{-\pi}{2}$ and $x = \frac{\pi}{2}$?

(a) $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sqrt{1 - \cos^2 x}\,dx.$

(b) $2\int_0^{\frac{\pi}{2}} \sqrt{1 + 2\sin^2 x}\,dx.$

(c) $\frac{\pi^2}{2}$

(d) $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sqrt{1 - \sin^2 x}\,dx.$

(e) $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sqrt{1 + \sin^2 x}\,dx.$

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5.(7 pts.) Consider the following sequences:

$$(I)\ \left\{(-1)^n\frac{n^2-1}{2^n}\right\}_{n=1}^{\infty} \qquad (II)\ \left\{(-1)^n\frac{n^2-1}{2n^2}\right\}_{n=1}^{\infty} \qquad (III)\ \left\{(-1)^n n\ln(n)\right\}_{n=1}^{\infty}$$

Which of the following statements is true?

(a) Sequences II and III converge but sequence I diverges.

(b) Sequence I converges but sequences II and III diverge.

(c) All three sequences diverge.

(d) Sequences I and II converge but sequence III diverges.

(e) All three sequences converge.

6.(7 pts.) Find the sum of the following series:

$$\sum_{n=1}^{\infty}\frac{(-1)^n 2^{n+1}}{3^n}.$$

(a) This series diverges. (b) $-\frac{4}{5}$ (c) $\frac{4}{5}$

(d) $-\frac{3}{5}$ (e) $\frac{3}{5}$

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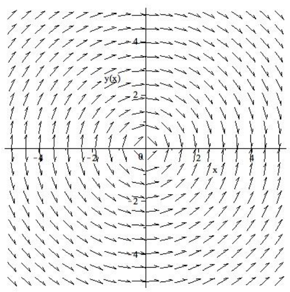
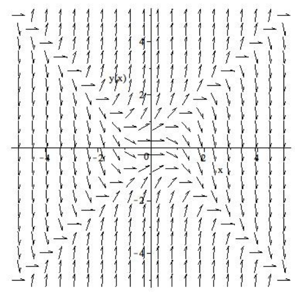
7.(7 pts.) Which of the following gives the direction field for the differential equation

$$y' = y^2 - x^2$$

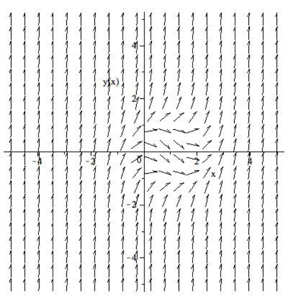
Note the letter corresponding to each graph is at the lower left of the graph.



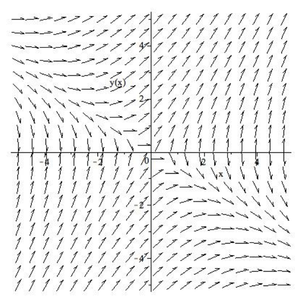
(a)



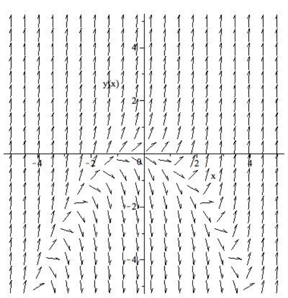
(b)



(c)



(d)



(e)

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8.(7 pts.) Use Euler's method with step size 0.1 to estimate $y(1.2)$ where $y(x)$ is the solution to the initial value problem

$$y' = xy + 1 \qquad y(1) = 0.$$

(a) $y(1.2) \approx .101$ (b) $y(1.2) \approx .112$ (c) $y(1.2) \approx .211$

(d) $y(1.2) \approx .201$ (e) $y(1.2) \approx .111$

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9.(7 pts.) Find the solution of the differential equation

$$\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{1+x^2}$$

with initial condition $y(0) = 0$.

(a) $y = \dfrac{x}{1+x}$ (b) $y = \dfrac{1}{\sqrt{1+x^2}}$ (c) $y = \dfrac{x}{\sqrt{1+x^2}}$

(d) $y = \dfrac{x^2}{\sqrt{1+x^2}}$ (e) $y = \dfrac{x}{1+x^2}$

10.(7 pts.) Find a general solution, valid for $-\frac{\pi}{2} < x < \frac{\pi}{2}$, of the differential equation

$$\frac{dy}{dx} - (\tan x)y = 1.$$

(a) $y = \dfrac{x + \sin x + C}{\cos x}$ (b) $y = \tan x + \cos x + C$ (c) $y = \dfrac{\cos x + C}{\sin x}$

(d) $y = \dfrac{\sin x + C}{\cos x}$ (e) $y = \dfrac{x + \sin x + C}{\sin x}$

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11.(7 pts.) Which of the following statements about the sequence

$$\left\{\frac{(\ln n)^2}{n}\right\}_{n=1}^{\infty}$$

is true?

(a) The sequence converges to e^2

(b) The sequence converges to ∞

(c) The sequence converges to 0

(d) The sequence diverges

(e) The sequence converges to 1

12.(7 pts.) Find the family of orthogonal trajectories to the family of curves given by

$$y = kx^2.$$

(a) $y = x^2 + C$

(b) $y = Cx^2$

(c) $y^2 - \frac{x^2}{2} = C$

(d) $y^2 + \frac{x^2}{2} = C$

(e) $y^2 - x^2 = C$

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Partial Credit

You must show your work on the partial credit problems to receive credit!

13.(10 pts.) Calculate the arc length of the curve if $y = \dfrac{x^2}{4} - \ln(\sqrt{x})$, where $2 \leq x \leq 4$.

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14. (12 pts.) Solve the initial value problem

$$xy' + xy + y = e^{-x}$$
$$y(1) = \frac{2}{e}.$$

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15. (6 pts.) Please circle "TRUE" if you think the statement is true, and circle "FALSE" if you think the statement is False.

(a)(1 pt. No Partial credit) The formula for the trapezoidal approximation with n approximating trapezoids is given by

$$\frac{\Delta x}{2}\left[2f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_n)\right].$$

TRUE FALSE

(b)(1 pt. No Partial credit) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ converges.

TRUE FALSE

(c))(1 pt. No Partial credit) $\displaystyle\int_1^{\infty} \frac{1}{x}$ diverges.

TRUE FALSE

(d))(1 pt. No Partial credit) If $\dfrac{dy}{dx} = \sqrt{x^2 + y^2}$ is a separable differential equation.

TRUE FALSE

(e))(1 pt. No Partial credit) The sequence $\left\{1 - \dfrac{1}{n}\right\}_{n=1}^{\infty}$ diverges.

TRUE FALSE

(f))(1 pt. No Partial credit) The slope field for the differential equation $\dfrac{dy}{dx} = xe^{-y}$ has a line with slope -1 at the point $(-1, 0)$.

TRUE FALSE

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The following is the list of useful trigonometric formulas:

Note: $\sin^{-1} x$ and $\arcsin(x)$ are different names for the same function and $\tan^{-1} x$ and $\arctan(x)$ are different names for the same function.

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin 2x = 2 \sin x \cos x$$

$$\sin x \cos y = \frac{1}{2}\big(\sin(x-y) + \sin(x+y)\big)$$

$$\sin x \sin y = \frac{1}{2}\big(\cos(x-y) - \cos(x+y)\big)$$

$$\cos x \cos y = \frac{1}{2}\big(\cos(x-y) + \cos(x+y)\big)$$

$$\int \sec\theta = \ln|\sec\theta + \tan\theta| + C$$

$$\int \csc\theta = \ln|\csc\theta - \cot\theta| + C$$

$$\csc\theta = \frac{1}{\sin\theta}, \quad \cot\theta = \frac{1}{\tan\theta}$$